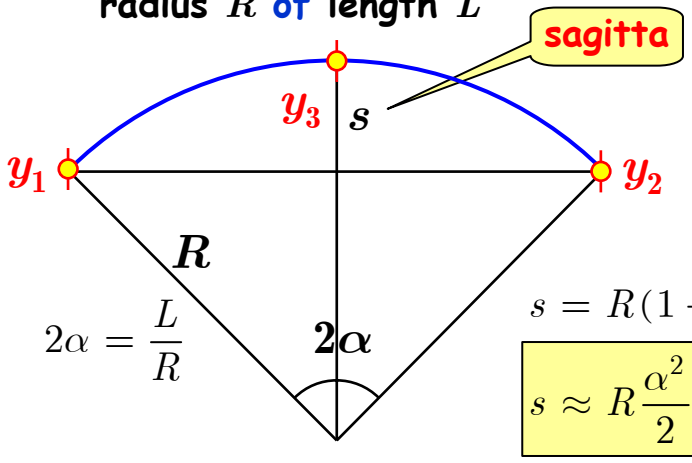
Momentum Measurement: Sagitta

- To introduce the problem of momentum measurement let's go back to the sagitta
- A particle moving in a plane perpendicular to a uniform magnetic field B

$$R = \frac{p}{0.3B} \qquad \frac{\delta p}{p} = \frac{\delta R}{R}$$

- The trajectory of the particle is an arc of radius R of length L



$$2\alpha = \frac{L}{R}$$

$$s = R(1-\cos\alpha)$$

$$s \approx R\frac{\alpha^2}{2} = \frac{L^2}{8R}$$

- Assume we have 3 measurements: y_1, y_2, y_3

$$s = y_3 - \frac{y_1 + y_2}{2} \qquad \delta s = \sqrt{\frac{3}{2}}\delta y \sim \delta y$$

- The error on the radius is related to the sagitta error by

$$|\delta s| = \frac{L^2}{8R}\frac{\delta R}{R} \sim \delta y \qquad \frac{L^2}{8R}\frac{\delta p}{p} = \delta y$$

$$\frac{\delta p}{p} = \frac{8R}{L^2}\delta y \qquad \frac{\delta p}{p} = \frac{8p}{0.3BL^2}\delta y$$

$$\frac{\delta p}{p^2} = \frac{8\delta y}{0.3BL^2}$$

- Important features
  - The percentage error on the momentum is proportional to the momentum itself
  - The error on the momentum is inversely proportional to B
  - The error on the momentum is inversely proportional to $1/L^2$
  - The error on the momentum is proportional to coordinate measurement error