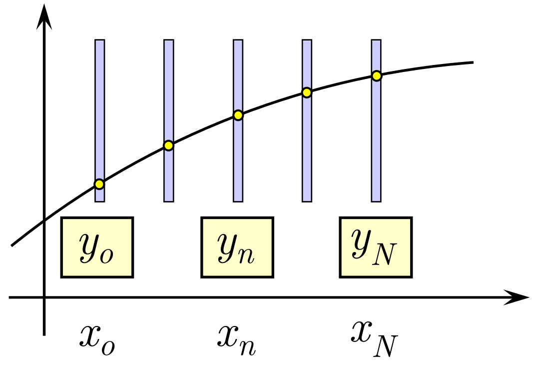
Quadratic Fit

- Assume N detectors measuring the y coordinate [Gluckstern 63]



- The detectors are placed at positions $x_0, \ldots, x_n, \ldots, x_N$
- A track crossing the detectors
  - Gives the measurements $y_0, \ldots, y_n, \ldots, y_N$
- Each measurement has an error σ_n
- Using the parabola approximation, the track parameters are found by minimizing the χ^2

$$\chi^2 = \sum_{n=0}^{N} \frac{\left(y_n - a - bx_n - cx_n^2\right)^2}{\sigma_n^2}$$

- However we can use the matrix formalism developed for the straight line:

$$\mathbf{Y} = \begin{pmatrix} y_o \\ \ldots \\ y_N \end{pmatrix} \quad \mathbf{p} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 1 & x_0 & x_0^2 \\ \ldots & \ldots & \ldots \\ 1 & x_N & x_N^2 \end{pmatrix} \quad \mathbf{W} = \begin{pmatrix} \frac{1}{\sigma_0^2} & 0 & 0 \\ 0 & \ldots & ..0 \\ 0 & 0 & \frac{1}{\sigma_N^2} \end{pmatrix}$$

- Let's recall the solution

$$\boxed{\tilde{\mathbf{p}} = \left(\mathbf{A}^T\mathbf{W}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{W}\mathbf{Y}}$$

$$\left(\mathbf{A}^T\mathbf{W}\mathbf{A}\right)^{-1} = \begin{pmatrix} F_0 & F_1 & F_2 \\ F_1 & F_2 & F_3 \\ F_2 & F_3 & F_4 \end{pmatrix}^{-1} \qquad F_k = \sum_{n=0}^{N} \frac{x_n^k}{\sigma_n^2}$$

$$\mathbf{A}^T\mathbf{W}\mathbf{Y} = \begin{pmatrix} M_0 \\ M_1 \\ M_2 \end{pmatrix} \qquad M_k = \sum_{n=0}^{N} \frac{y_n x_n^k}{\sigma_n^2}$$