HighRR TFR Hands-On: performances

Alessio Piucci

1 June 2017
HighRR TFR Hands-On

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

HighRR

Performances

At this point, you have:

- **1D chi2 linear fit;**
- **1D chi2 quadratic fit;**
- **Kalman filtering.**

On which bases the methods can be compared?

Performances

- momentum resolution
- vertex resolution (at z = 0)
- track quality (χ^2)
- timing

Many scenarios to check:

- switch on/off the multiple scattering
- detector layers in/out B
- modify the lever arm and B
- ...

Vertex extrapolation

Kalman fit: already there

x^2 fit: precision can be improved with 2D fit

→ let's implement a 2D x^2 fit for straight lines

Vertex extrapolation - x^2

1D fit case: only fit x coordinates (z are fixed by the geometry…)

$$A = \begin{pmatrix} 1 & z_0 \\ 1 & z_1 \\ \dots & \dots \\ 1 & z_n \end{pmatrix} \qquad X = \begin{pmatrix} x_0 \\ x_1 \\ \dots \\ x_n \end{pmatrix} \qquad V = \begin{pmatrix} \sigma_0^2 & 0 & 0 & 0 \\ 0 & \sigma_1^2 & 0 & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & \sigma_n^2 \end{pmatrix} \qquad P = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$P = V * A^t * V^{-1} * X$$
$$V_p = (A^t * W * A)^{-1}$$

Vertex extrapolation - χ^2

2D fit case: fit x and y together

$$A = \begin{pmatrix} 1 & z_0 & 0 & 0 \\ 0 & 0 & 1 & z_0 \\ 1 & z_1 & 0 & 0 \\ 0 & 0 & 1 & z_1 \\ \dots & \dots & \dots & \dots \\ 1 & z_n & 0 & 0 \\ 0 & 0 & 1 & z_n \end{pmatrix} \quad X = \begin{pmatrix} x_0 \\ y_0 \\ x_1 \\ y_1 \\ \dots \\ x_n \\ y_n \end{pmatrix} \quad V = \begin{pmatrix} \sigma_{x0}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{y0}^2 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \sigma_{xn}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{yn}^2 \end{pmatrix} \quad P = \begin{pmatrix} a_x \\ b_x \\ a_y \\ b_y \end{pmatrix}$$

$$P = V * A^t * V^{-1} * X$$

$$V_p = (A^t * W * A)^{-1}$$

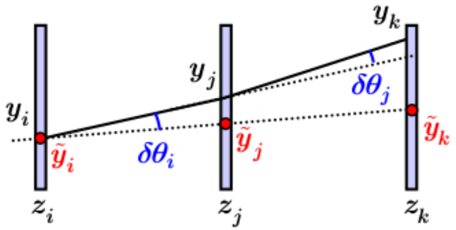
Tasks

1. Implement the extrapolation to z=0 for chi2 (this is the 2D fit)
2. Remove the B field from the geometry (set z-start behind the detector) and compare vertex position/chi2/p_value for Kalman and Chi2
   - From this they will see that in the linear case without multiple scattering Kalman and chi2 give exactly the same results
3. Switch on B_field in the second halve of the detector and compare momentum resolution/chi2 for Kalman and Chi2
   - From this they will see that an imperfect model in the chi2 fit leads to high chi2 values and a poor momentum resolution/ a bias in comparison to the Kalman
4. Turn on multiple scattering and go back to 2.
   - From this they will see that the chi2 fit (which neglects MS) has poor chi2 and a poor vertex resolution in comparison to the Kalman
5. Turn on multiple scattering and go back to 3.
   - From this they will see: Also momentum resolution is affected by multiple scattering

Multiple scattering

Multiple Scattering

- Multiple scattering is a cumulative effect and introduces correlation among the coordinate measurements
- The treatment of multiple scattering is different for:
  - Discrete detectors
  - Continuous detectors
- Here we consider only the simplest case of discrete and thin detectors
- For continuous detectors see for example [5: Avery 1991]
- Let's consider 3 thin detectors



- A track cross the 3 planes at positions

$$\tilde{y}_i \quad \tilde{y}_j \quad \tilde{y}_k$$

- The 3 coordinate have measurement errors σ_i, σ_j, σ_k due to the detector resolution
- They also have mean value

$$\boxed{\langle \tilde{y}_i \rangle = \bar{\tilde{y}}_i \quad \langle \tilde{y}_j \rangle = \bar{\tilde{y}}_j \quad \langle \tilde{y}_k \rangle = \bar{\tilde{y}}_k}$$

- On plane i $\quad y_i = \tilde{y}_i$
- Because of multiple scattering on plane i the actual trajectory cross plane j at

$$y_j = \tilde{y}_j + (z_j - z_i)\delta\theta_i$$

- Because of multiple scattering on planes i,j the actual trajectory cross plane k at

$$y_k = \tilde{y}_k + (z_k - z_i)\delta\theta_i + (z_k - z_j)\delta\theta_j$$

- Since $\quad \langle \delta\theta \rangle = 0$

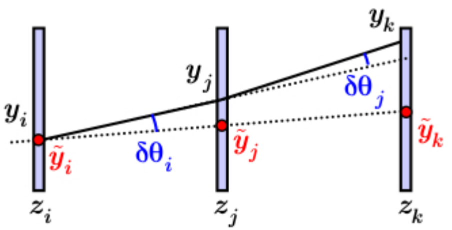
$$\bar{y}_i = \bar{\tilde{y}}_i \qquad \bar{y}_j = \bar{\tilde{y}}_j \qquad \bar{y}_k = \bar{\tilde{y}}_k$$

Multiple scattering

Multiple Scattering

- we can now compute the covariance matrix of the coordinate measurements including multiple scattering

$$V_{nm} = \langle (y_m - \bar{y}_m)(y_n - \bar{y}_n) \rangle$$



- First the diagonal elements

$$V_{ii} = \left\langle (y_i - \bar{y}_i)^2 \right\rangle = \left\langle (y_i - \bar{\tilde{y}}_i)^2 \right\rangle$$

$$\boxed{V_{ii} = \sigma_i^2}$$

$$V_{jj} = \left\langle (y_j - \bar{y}_j)^2 \right\rangle = \left\langle \left(y_j - \bar{\tilde{y}}_j + (z_j - z_i)\delta\theta_i\right)^2 \right\rangle$$

$$= \left\langle (y_j - \bar{\tilde{y}}_j)^2 \right\rangle + (z_j - z_i)^2 \left\langle \delta\theta_i^2 \right\rangle \cancel{+ 2(z_j - z_i)\left\langle (y_j - \bar{\tilde{y}}_j)\delta\theta_i \right\rangle}$$

$$\boxed{V_{jj} = \sigma_j^2 + (z_j - z_i)^2 \left\langle \delta\theta_i^2 \right\rangle}$$

- It easy to verify that

$$\boxed{V_{kk} = \sigma_k^2 + (z_k - z_i)^2 \left\langle \delta\theta_i^2 \right\rangle + (z_k - z_j)^2 \left\langle \delta\theta_j^2 \right\rangle}$$

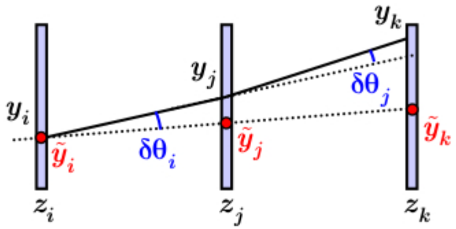
$$\langle \delta\theta^2 \rangle = \sigma_{ms}$$

An Introduction to Charged Particles Tracking - Francesco Ragusa 37

Multiple scattering

Multiple Scattering

$$V_{nm} = \langle (y_m - \bar{y}_m)(y_n - \bar{y}_n) \rangle$$



- The off-diagonal elements are

$$V_{ij} = \langle (y_i - \bar{y}_i)(y_j - \bar{y}_j) \rangle = \langle (\underbrace{y_i - \bar{\tilde{y}}_i}) (\underbrace{y_j - \bar{\tilde{y}}_j} + \underbrace{(z_j - z_i)\delta\theta_i}) \rangle \qquad \boxed{V_{ij} = 0}$$

uncorrelated: <>=0

$$V_{ik} = \langle (y_i - \bar{y}_i)(y_k - \bar{y}_k) \rangle$$

$$= \langle (\underbrace{y_i - \bar{\tilde{y}}_i})(\underbrace{y_k - \bar{\tilde{y}}_k} + \underbrace{(z_k - z_i)\delta\theta_i} + \underbrace{(z_k - z_j)\delta\theta_j}) \rangle \qquad \boxed{V_{ik} = 0}$$

$$V_{jk} = \langle (y_j - \bar{y}_j)(y_k - \bar{y}_k) \rangle$$

uncorrelated: <>=0

$$= \langle (y_j - \bar{\tilde{y}}_j + (z_j - z_i)\delta\theta_i)(y_k - \bar{\tilde{y}}_k + (z_k - z_i)\delta\theta_i + (z_k - z_j)\delta\theta_j) \rangle$$

$$= \langle (z_j - z_i)\delta\theta_i (z_k - z_i)\delta\theta_i \rangle$$

$$\boxed{V_{jk} = (z_j - z_i)(z_k - z_i)\langle \delta\theta_i^2 \rangle}$$

An Introduction to Charged Particles Tracking - Francesco Ragusa 38

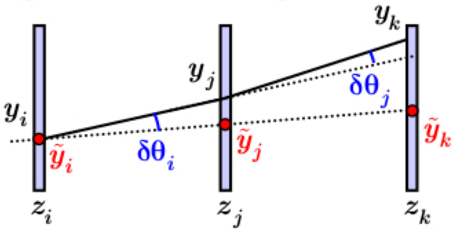
Multiple scattering

Multiple Scattering

- Summarizing, the covariance matrix is

$$V = \begin{pmatrix} \sigma_i^2 & 0 & 0 \\ 0 & \sigma_j^2 & 0 \\ 0 & 0 & \sigma_k^2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & (z_j - z_i)^2 \delta\theta_i^2 & (z_k - z_i)(z_j - z_i)\delta\theta_i^2 \\ 0 & (z_k - z_i)(z_j - z_i)\delta\theta_i^2 & (z_k - z_i)^2 \delta\theta_i^2 + (z_j - z_i)^2 \delta\theta_j^2 \end{pmatrix}$$

- The second matrix has
  - Diagonal elements due to any previous material affecting the trajectory impact point at the given plane
  - Off diagonal elements: only presents if a previous material layer affects at the same time the trajectory impact points for the 2 planes
- The same scattering at plane i
- Affects the trajectory at plane j and plane k



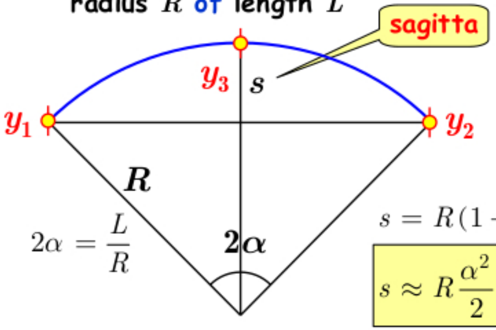
Momentum resolution

Momentum Measurement: Sagitta

- To introduce the problem of momentum measurement let's go back to the sagitta
- A particle moving in a plane perpendicular to a uniform magnetic field B

$$\boxed{R = \frac{p}{0.3B} \qquad \frac{\delta p}{p} = \frac{\delta R}{R}}$$

- The trajectory of the particle is an arc of radius R of length L



$$2\alpha = \frac{L}{R}$$

$$s = R(1 - \cos\alpha)$$

$$\boxed{s \approx R\frac{\alpha^2}{2} = \frac{L^2}{8R}}$$

- Assume we have 3 measurements: y_1, y_2, y_3

$$s = y_3 - \frac{y_1 + y_2}{2} \qquad \delta s = \sqrt{\frac{3}{2}}\delta y \sim \delta y$$

- The error on the radius is related to the sagitta error by

$$|\delta s| = \frac{L^2}{8R}\frac{\delta R}{R} \sim \delta y \qquad \frac{L^2}{8R}\frac{\delta p}{p} = \delta y$$

$$\frac{\delta p}{p} = \frac{8R}{L^2}\delta y \qquad \frac{\delta p}{p} = \frac{8p}{0.3BL^2}\delta y$$

$$\boxed{\frac{\delta p}{p^2} = \frac{8\delta y}{0.3BL^2}}$$

- Important features
  - The percentage error on the momentum is proportional to the momentum itself
  - The error on the momentum is inversely proportional to B
  - The error on the momentum is inversely proportional to $1/L^2$
  - The error on the momentum is proportional to coordinate measurement error

An Introduction to Charged Particles Tracking - Francesco Ragusa 20

Momentum resolution with MS

Track Fit With Multiple Scattering

- The methods developed to fit a track to the measured points can be used to perform a fit taking into account M.S.
  - The covariance matrix is computed
  - The same fit procedure is applied
- However the calculation is quite long.
  - In the Gluckstern paper an example is worked and assuming
    - Error dominated by Multiple Scattering
    - Equally spaced points (> 3)

$$\boxed{\frac{\delta p}{p} \sim \frac{1}{0.3B}\frac{0.0136}{\beta}\sqrt{\frac{1.3}{X_0 L}}}$$

- We note that
  - The resolution does not depend on momentum anymore
  - We still have a dependence on the particle velocity
- In this example the spatial resolution is dominated by Multiple Scattering
  - Since Multiple Scattering diminish with momentum p there is always a minimum momentum above which the assumption is not true and we change regime

An Introduction to Charged Particles Tracking - Francesco Ragusa 40

Tasks

1) Task: implement the multiple scattering in the covariance of the x^2 fit

2) Task: performances with MS!

- **verify the vertex resolution with/without MS, without B field at all (config_geo_allpixel_noBatAll.info): MS taken in account in both chi2 and Kalman**

- **verify the momentum resolution with/without MS, with B between the two sub-detectors (config_geo_allpixel.info): MS taken in account *in Kalman only***