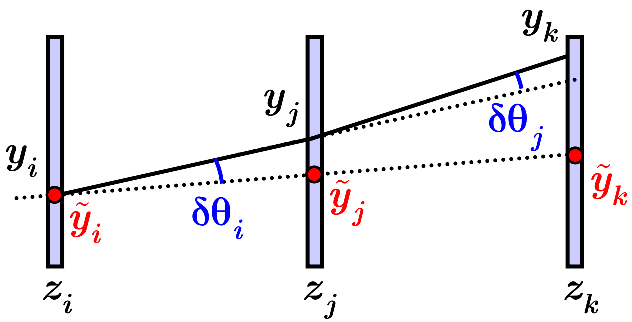
Multiple Scattering

- we can now compute the **covariance matrix** of the coordinate measurements including multiple scattering

$$V_{nm} = \langle (y_m - \bar{y}_m)(y_n - \bar{y}_n) \rangle$$



- First the diagonal elements

$$V_{ii} = \left\langle (y_i - \bar{y}_i)^2 \right\rangle = \left\langle (y_i - \bar{\tilde{y}}_i)^2 \right\rangle$$

$$\boxed{V_{ii} = \sigma_i^2}$$

$$V_{jj} = \left\langle \left(y_j - \bar{y}_j\right)^2 \right\rangle = \left\langle \left(y_j - \bar{\tilde{y}}_j + \left(z_j - z_i\right)\delta\theta_i\right)^2 \right\rangle$$

$$= \left\langle \left(y_j - \bar{\tilde{y}}_j\right)^2 \right\rangle + \left(z_j - z_i\right)^2 \left\langle \delta\theta_i^2 \right\rangle \cancel{+2\left(z_j - z_i\right)\left\langle \left(y_j - \bar{\tilde{y}}_j\right)\delta\theta_i \right\rangle}$$

$$\boxed{V_{jj} = \sigma_j^2 + \left(z_j - z_i\right)^2 \left\langle \delta\theta_i^2 \right\rangle}$$

- It easy to verify that

$$\boxed{V_{kk} = \sigma_k^2 + \left(z_k - z_i\right)^2 \left\langle \delta\theta_i^2 \right\rangle + \left(z_k - z_j\right)^2 \left\langle \delta\theta_j^2 \right\rangle}$$