HighRR TFR Hands-On: x^2 stuff – Episode 2

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HighRR TFR Hands-On

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χ^2 distribution

Previously, on this channel: chi2 fit for straight lines

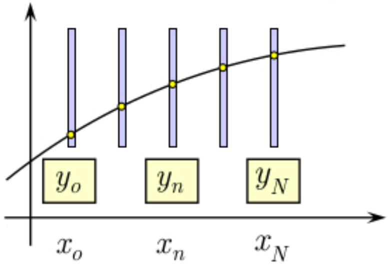
If the detector is in uniform B → circular trajectory

You may don’t want to fit a circular parametrisation...

x^2 minimisation

Quadratic Fit

- Assume N detectors measuring the y coordinate [Gluckstern 63]



- The detectors are placed at positions $x_0, \ldots, x_n, \ldots, x_N$
- A track crossing the detectors
  - Gives the measurements $y_0, \ldots, y_n, \ldots, y_N$
- Each measurement has an error σ_n
- Using the parabola approximation, the track parameters are found by minimizing the χ^2

$$\chi^2 = \sum_{n=0}^{N} \frac{\left(y_n - a - bx_n - cx_n^2\right)^2}{\sigma_n^2}$$

- However we can use the matrix formalism developed for the straight line:

$$\mathbf{Y} = \begin{pmatrix} y_o \\ \ldots \\ y_N \end{pmatrix} \quad \mathbf{p} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} 1 & x_0 & x_0^2 \\ \ldots & \ldots & \ldots \\ 1 & x_N & x_N^2 \end{pmatrix} \quad \mathbf{W} = \begin{pmatrix} \frac{1}{\sigma_0^2} & 0 & 0 \\ 0 & \ldots & ..0 \\ 0 & 0 & \frac{1}{\sigma_N^2} \end{pmatrix}$$

- Let's recall the solution

$$\boxed{\tilde{\mathbf{p}} = \left(\mathbf{A}^T\mathbf{W}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{W}\mathbf{Y}}$$

$$\left(\mathbf{A}^T\mathbf{W}\mathbf{A}\right)^{-1} = \begin{pmatrix} F_0 & F_1 & F_2 \\ F_1 & F_2 & F_3 \\ F_2 & F_3 & F_4 \end{pmatrix}^{-1} \qquad F_k = \sum_{n=0}^{N} \frac{x_n^k}{\sigma_n^2}$$

$$\mathbf{A}^T\mathbf{W}\mathbf{Y} = \begin{pmatrix} M_0 \\ M_1 \\ M_2 \end{pmatrix} \qquad M_k = \sum_{n=0}^{N} \frac{y_n x_n^k}{\sigma_n^2}$$

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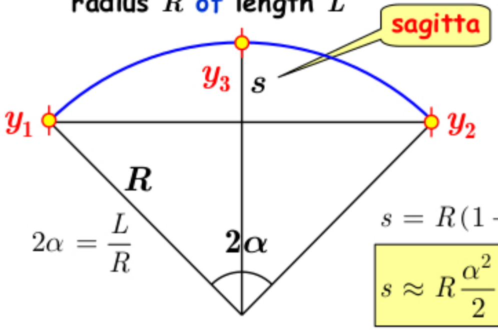
Momentum measurement

Momentum Measurement: Sagitta

- To introduce the problem of momentum measurement let's go back to the sagitta
- A particle moving in a plane perpendicular to a uniform magnetic field B

$$\boxed{R = \frac{p}{0.3B} \qquad \frac{\delta p}{p} = \frac{\delta R}{R}}$$

- The trajectory of the particle is an arc of radius R of length L



$$2\alpha = \frac{L}{R}$$

$$s = R(1 - \cos\alpha)$$

$$\boxed{s \approx R\frac{\alpha^2}{2} = \frac{L^2}{8R}}$$

- Assume we have 3 measurements: y_1, y_2, y_3

$$s = y_3 - \frac{y_1 + y_2}{2} \qquad \delta s = \sqrt{\frac{3}{2}}\delta y \sim \delta y$$

- The error on the radius is related to the sagitta error by

$$|\delta s| = \frac{L^2}{8R}\frac{\delta R}{R} \sim \delta y \qquad \frac{L^2}{8R}\frac{\delta p}{p} = \delta y$$

$$\frac{\delta p}{p} = \frac{8R}{L^2}\delta y \qquad \frac{\delta p}{p} = \frac{8p}{0.3BL^2}\delta y$$

$$\boxed{\frac{\delta p}{p^2} = \frac{8\delta y}{0.3BL^2}}$$

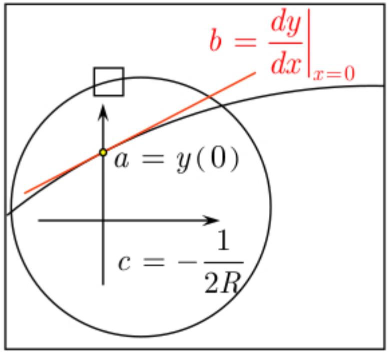
- Important features
  - The percentage error on the momentum is proportional to the momentum itself
  - The error on the momentum is inversely proportional to B
  - The error on the momentum is inversely proportional to $1/L^2$
  - The error on the momentum is proportional to coordinate measurement error

Momentum measurement

Tracking In Magnetic Field

- The previous example showed the basic features of momentum measurement
- Let's now turn to a more complete treatment of the measurement of the charged particle trajectory
- We have already seen that for an homogeneous magnetic field the trajectory projected on a plane perpendicular to the magnetic field is a circle

$$(y - y_o)^2 + (x - x_o)^2 = R^2$$



- For not too low momenta we can use a linear approximation

$$y = y_o + \sqrt{R^2 - (x - x_o)^2}$$

$$R^2 \gg (x - x_o)^2$$

$$y \approx y_o + R\left(1 - \frac{(x - x_o)^2}{2R^2}\right)$$

$$y = \left(y_o + R - \frac{x_o^2}{2R}\right) + \frac{x_o}{R}x - \frac{1}{2R}x^2$$

- We are led to the parabolic approximation of the trajectory

$$\boxed{y = a + bx + cx^2}$$

- Let's stress that as far as the track parameters is concerned the dependence is linear
- The parameters a,b,c are
  - Intercept at the origin
  - Slope at the origin
  - Radius of curvature (momentum)

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χ^2 minimisation - Tasks

1) implement a 1D chi2 fit for parabolic trajectories
in the TFRChiSquaredFit class

- do not take in account multiple scattering in the covariance
- start with a weak B field...

Input: clusters of a reconstructed track
Output: estimation of a, b, c parameters

2) fit the complete track with straight tracklet + parabola

- put some detector layers in B

3) measure the momentum from R = -1./2c