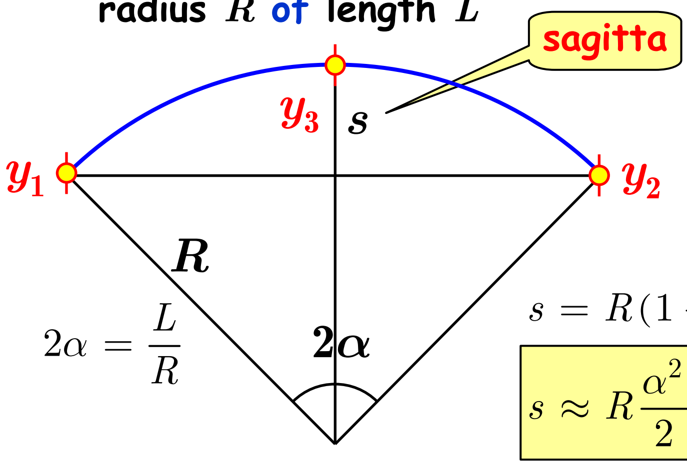
Momentum Measurement: Sagitta

- To introduce the problem of momentum measurement let's go back to the sagitta
- A particle moving in a plane perpendicular to a uniform magnetic field B

$$\boxed{R = \frac{p}{0.3B} \qquad \frac{\delta p}{p} = \frac{\delta R}{R}}$$

- The trajectory of the particle is an arc of radius R of length L



$$2\alpha = \frac{L}{R}$$

$$s = R(1-\cos\alpha)$$

$$\boxed{s \approx R\frac{\alpha^2}{2} = \frac{L^2}{8R}}$$

- Assume we have 3 measurements: y_1, y_2, y_3

$$s = y_3 - \frac{y_1+y_2}{2} \qquad \delta s = \sqrt{\frac{3}{2}}\delta y \sim \delta y$$

- The error on the radius is related to the sagitta error by

$$|\delta s| = \frac{L^2}{8R}\frac{\delta R}{R} \sim \delta y \qquad \frac{L^2}{8R}\frac{\delta p}{p} = \delta y$$

$$\frac{\delta p}{p} = \frac{8R}{L^2}\delta y \qquad \frac{\delta p}{p} = \frac{8p}{0.3BL^2}\delta y$$

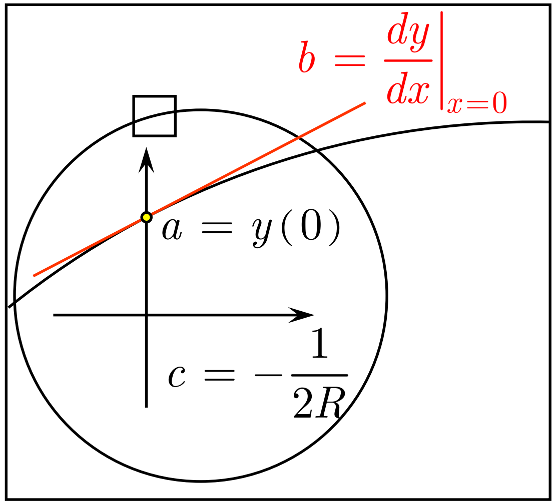
$$\boxed{\frac{\delta p}{p^2} = \frac{8\delta y}{0.3BL^2}}$$

- Important features
  - The percentage error on the momentum is proportional to the momentum itself
  - The error on the momentum is inversely proportional to B
  - The error on the momentum is inversely proportional to $1/L^2$
  - The error on the momentum is proportional to coordinate measurement error

Tracking In Magnetic Field

- The previous example showed the basic features of momentum measurement
- Let's now turn to a more complete treatment of the measurement of the charged particle trajectory
- We have already seen that for an homogeneous magnetic field the trajectory projected on a plane perpendicular to the magnetic field is a circle

$$(y - y_o)^2 + (x - x_o)^2 = R^2$$



- For not too low momenta we can use a linear approximation

$$y = y_o + \sqrt{R^2 - (x - x_o)^2}$$

$$\boxed{R^2 \gg (x - x_o)^2}$$

$$y \approx y_o + R\left(1 - \frac{(x - x_o)^2}{2R^2}\right)$$

$$y = \left(y_o + R - \frac{x_o^2}{2R}\right) + \frac{x_o}{R}x - \frac{1}{2R}x^2$$

- We are led to the parabolic approximation of the trajectory

$$\boxed{y = a + bx + cx^2}$$

- Let's stress that as far as the track parameters is concerned the dependence is linear
- The parameters a,b,c are
  - Intercept at the origin
  - Slope at the origin
  - Radius of curvature (momentum)