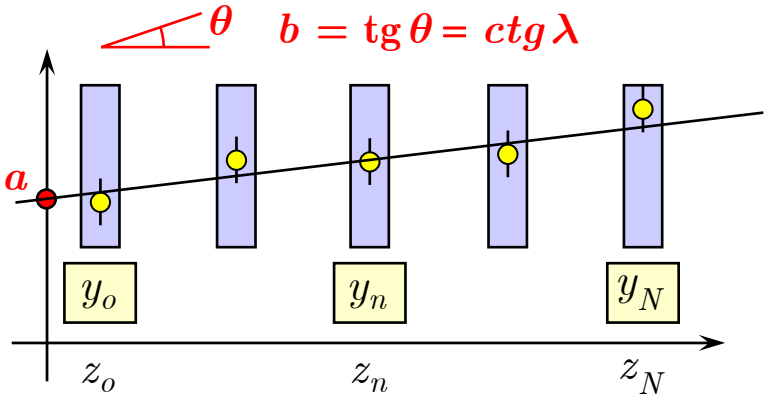
Straight Line Fit

- This is a well known problem
  - A reference frame
  - $N+1$ measuring detetectors at $z_0,\ldots, z_n, \ldots, z_N$
  - A particle crossing the detectors
  - $N+1$ coordinate measurements $y_0,\ldots, y_n, \ldots, y_N$
  - Each measurement affected by uncorrelated errors $\sigma_0,\ldots, \sigma_n, \ldots, \sigma_N$
- Find the best line $y = a + b\ z$ that fit the track
- The solution is found by minimizing the χ^2



$$\chi^2 = \sum_{n=0}^{N} \frac{(y_n - a - bz_n)^2}{\sigma_n^2}$$

$$a = (S_y S_{zz} - S_z S_{zy})/D$$
$$b = (S_1 S_{zy} - S_z S_y)/D$$

$$S_1 = \sum_{n=0}^{N} \frac{1}{\sigma_n^2} \qquad S_y = \sum_{n=0}^{N} \frac{y_n}{\sigma_n^2}$$

$$S_z = \sum_{n=0}^{N} \frac{z_n}{\sigma_n^2} \qquad S_{yz} = \sum_{n=0}^{N} \frac{y_n z_n}{\sigma_n^2}$$

$$S_{zz} = \sum_{n=0}^{N} \frac{z_n^2}{\sigma_n^2} \qquad D = S_1 S_{zz} - S_z S_z$$

- The covariance matrix (at $z = 0$) is

$$\begin{pmatrix} \sigma_a^2 & c_{ab} \\ c_{ab} & \sigma_b^2 \end{pmatrix} = \frac{1}{D} \begin{pmatrix} S_{zz} & -S_z \\ -S_z & S_1 \end{pmatrix}$$

Depends only on σ, z_n and N