Straight Line Fit: Matrix Formalism

- It is useful to restate the problem using a matrix formalism [4:Avery 1991]
- This is useful because:
  - It is more compact
  - It is easily extensible to other linear problems
  - It is more useful to formulate an iterative procedure
- With the same assumption as before
  - The linear model is given by $\mathbf{f} = \mathbf{A}\mathbf{p}$

$$\mathbf{f} = \begin{pmatrix} f_0 \\ \dots \\ f_N \end{pmatrix} = \begin{pmatrix} a + bz_0 \\ \\ a + bz_N \end{pmatrix} = \begin{pmatrix} 1 & z_0 \\ 1 & \dots \\ 1 & z_N \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \mathbf{A}\mathbf{p}$$

  - Measurements and errors are

$$\mathbf{Y} = \begin{pmatrix} y_0 \\ \dots \\ y_N \end{pmatrix} \qquad (\mathbf{V})_{ij} = \left\langle \left(y_i - \langle y_i \rangle\right)\left(y_j - \langle y_j \rangle\right) \right\rangle$$

$$(\mathbf{V})_{ij} = \sigma_i^2 \delta_{ij} \quad \boxed{\text{if uncorrelated}}$$

- The χ^2 can be written as

$$\chi^2 = (\mathbf{Y} - \mathbf{A}\mathbf{p})^T \mathbf{W} (\mathbf{Y} - \mathbf{A}\mathbf{p}) \qquad \mathbf{W} = \mathbf{V}^{-1}$$

- The minimum χ^2 is obtained by

$$\boxed{\tilde{\mathbf{p}} = \left(\mathbf{A}^T \mathbf{W} \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{W} \mathbf{Y}}$$

- The covariance matrix of the parameters is obtained from covariance matrix V of the measurements

$$\boxed{\mathbf{V}_{\mathbf{p}} = \left(\mathbf{A}^T \mathbf{V}^{-1} \mathbf{A}\right)^{-1}}$$

- Please notice (N+1 measurements, M parameters)
  - Dimensions $\mathbf{A} = (N+1) \times M$
  - Dimensions $\mathbf{V} = (N+1) \times (N+1)$
  - Dimensions $\mathbf{A}^T\mathbf{W}\mathbf{A} = M \times M$
  - Dimensions $\mathbf{A}^T\mathbf{W} = M \times (N+1)$
  - Dimensions $\mathbf{V}_\mathbf{p} = M \times M$