

Definitions

Strip 1 end and start: \vec{a} and \vec{b}

Strip 2 end and start: \vec{c} and \vec{d}

$$\vec{q} \equiv \vec{a} - \vec{b}$$

$$\vec{r} \equiv \vec{c} - \vec{d}$$

$$\vec{s} \equiv \vec{a} + \vec{b}$$

$$\vec{t} \equiv \vec{c} + \vec{d}$$

Perpendicular

$$\begin{pmatrix} q_1 & r_1 \\ q_2 & r_2 \end{pmatrix} \begin{pmatrix} m \\ -n \end{pmatrix} = \begin{pmatrix} t_1 - s_1 \\ t_2 - s_2 \end{pmatrix} \Rightarrow \begin{pmatrix} m \\ -n \end{pmatrix} = \frac{\begin{pmatrix} r_2 & -r_1 \\ -q_2 & q_1 \end{pmatrix}}{\begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix}} \begin{pmatrix} t_1 - s_1 \\ t_2 - s_2 \end{pmatrix}$$

Vertex constrained

$$\frac{1}{2} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} + m \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = k \left(\frac{1}{2} \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} + n \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \right) - \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$\frac{m}{2} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} - \frac{kn}{2} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} - k \begin{pmatrix} t_1/2 - v_1 \\ t_2/2 - v_2 \\ t_3/2 - v_3 \end{pmatrix} = - \begin{pmatrix} s_1/2 - v_1 \\ s_2/2 - v_2 \\ s_3/2 - v_3 \end{pmatrix}$$

Let

$$\vec{u} = \vec{s}/2 - \vec{v}$$

$$\vec{w} = \vec{t}/2 - \vec{v}$$

These are vectors from the vertex to the midpoint of the strips.

$$\begin{pmatrix} w_1 & q_1 & r_1 \\ w_2 & q_2 & r_2 \\ w_3 & q_3 & r_3 \end{pmatrix} \begin{pmatrix} -k \\ m/2 \\ -kn/2 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

Multiply from the left by the inverse of the 3×3 matrix:

$$\begin{pmatrix} -k \\ m/2 \\ -kn/2 \end{pmatrix} = \frac{\begin{pmatrix} (\vec{q} \times \vec{r})_1 & (\vec{q} \times \vec{r})_2 & (\vec{q} \times \vec{r})_3 \\ (\vec{r} \times \vec{w})_1 & (\vec{r} \times \vec{w})_2 & (\vec{r} \times \vec{w})_3 \\ (\vec{q} \times \vec{w})_1 & (\vec{q} \times \vec{w})_2 & (\vec{q} \times \vec{w})_3 \end{pmatrix}}{\begin{vmatrix} w_1 & q_1 & r_1 \\ w_2 & q_2 & r_2 \\ w_3 & q_3 & r_3 \end{vmatrix}} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

Hence

$$m/2 = \frac{(\vec{u} \cdot (\vec{r} \times \vec{w}))}{\begin{vmatrix} w_1 & q_1 & r_1 \\ w_2 & q_2 & r_2 \\ w_3 & q_3 & r_3 \end{vmatrix}}$$

$$x = \frac{1}{2}(m \vec{q} + \vec{s})$$